Due Date: April 5th, 2012

1. `glycerin.dat` provides data values of viscosity of glycerine versus temperature. Write a function file `newcst` that returns the viscosity of glycerine as a function of temperature. The program should evaluate a cubic polynomial in a Newton Basis based on data at temperatures $10, 20$, and 30 degrees. You should use the `divDiffTable` in the NMM tool box to compute coefficients of your polynomial, store the values of these coefficients as a vector and then evaluate the Newton polynomial.

2. Consider the following data set between variables x and y:

x	1986	1988	1990	1992	1994	1996
y	113.5	132.2	138.7	141.5	137.6	144.2

   (a) Creating an approrpriate Vandermonde matrix using the `vander` commmand, find the 5-th degree polynomial interpolating the data. Find the condition number of the Vandermonde matrix. Plot it.

   (b) Using `lagrint` function in NMM toolbox, find the coefficients of the 5-th degree polynomial using Lagrange basis. See if there is any difference with (a).

3. Following data set between variables x and y:

x	0.4	0.75	1.3	2
y	4.95	10.14	15	17.6

   (a) Using `divDiffTable` construct the divided difference table.

   (b) Extract the coefficients of the Newton Polynomial

4. The `H2Osat.dat` file in the data directory of the NMM toolbox contains saturation data for water. Use the `divDiffTable` function to construct the divided-difference table and extract the coefficients of the Newton interpolating polynomial in the range $30 \leq T \leq 35$.

5. Consider the following data set:

x	y
1	1
2	3
3	2
4	4

   Modify `splintFE` to determine the coefficients of the cubic-spline interpolant with zero Fixed-Slope End conditions and plot this spline between this range.

6. Consider $y = xe^{-x}$, for $0 \leq x \leq 8$. Write a function file, using `hermint` , that creates a piecewise-cubic Hermite approximations with $4, 6, 8, 12$ equally spaced points. Plot all 4 of these curves and the function on 4 different graphs.

7. Find the cubic-spline passing through $(x, y) = (1, 1), (2, 3), (3, 2)$ and $(4, 4)$. and having zero slope at $x = 1$ and $x = 4$ using `splintFE`. Plot the spline.

8. Show the following Theorem for $n = 2$ case.

Theorem:*Assume that $f \in C^n([a,b])$ and that $x_1, x_2, \dots x_n \in [a,b]$ are n nodes. If $x \in [a,b]$, then*

$$f(x) = P_{n-1}(x) + e_{n-1}(x),$$

where P_{n-1} is the Lagrange Polynomial of order $n-1$ and

$$e_{n-1}(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)f^n(c)}{n!}$$

for some value $c \equiv c(x)$.

9. Let $(x_i, f(x_i), f'(x_i)), i = 1, \dots n$ be given. Let

$$P_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3,$$

be the Hermite cubic interpolant in the range $[x_i, x_{i+1}]$. Show that the following constraints:

$$P_i(x_i) = f(x_i), P_i'(x_i) = f'(x_i), P_i(x_{i+1}) = f(x_{i+1}), P_i'(x_{i+1}) = f'(x_{i+1}), 1 \le i \le n-1,$$

imply that

$$\begin{aligned} a_i &= f(x_i), \\ b_i &= f'(x_i), \\ c_i &= \frac{3f[x_i, x_{i+1}] - 2f'(x_i) - f'(x_{i+1})}{(x_{i+1} - x_i)} \\ d_i &= \frac{f'(x_i) - 2f[x_i, x_{i+1}] + f'(x_{i+1})}{(x_{i+1} - x_i)^2} \end{aligned}$$

10. Complete the proof of cubic splines outlined in class for all the three end conditions : (i) Fixed-Slope (ii) Natural and (iii) Not a Knot