1. Consider the following data set:

x	y
1	1
2	3
3	2
4	4

   Modify `splintFE` to determine the coefficients of the cubic-spline interpolant with zero Fixed-Slope End conditions and plot this spline between this range.

2. Consider $y = xe^{-x}$, for $0 \leq x \leq 8$. Write a function file, using `hermint` , that creates a piecewise-cubic Hermite approximations with $4, 6, 8, 12$ equally spaced points. Plot all 4 of these curves and the function on 4 different graphs.

3. Find the cubic-spline passing through $(x, y) = (1, 1), (2, 3), (3, 2)$ and $(4, 4)$. and having zero slope at $x = 1$ and $x = 4$ using `splintFE`. Plot the spline.

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1. Show the following Theorem for $n = 2$ case.

   Theorem:*Assume that $f \in C^n([a, b])$ and that $x_1, x_2, \ldots x_n \in [a, b]$ are n nodes. If $x \in [a, b]$, then*

$$f(x) = P_{n-1}(x) + e_{n-1}(x),$$

   where P_{n-1} is the Lagrange Polynomial of order $n - 1$ and

$$e_{n-1}(x) = \frac{(x - x_1)(x - x_2) \ldots (x - x_n) f^n(c)}{n!}$$

   for some value $c \equiv c(x)$.

2. Let $(x_i, f(x_i), f'(x_i)), i = 1, \ldots n$ be given. Let

$$P_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3,$$

   be the Hermite cubic interpolant in the range $[x_i, x_{i+1}]$. Show that under the following constraints:

$$P_i(x_i) = f(x_i), P_i'(x_i) = f'(x_i), P_i(x_{i+1}) = f(x_{i+1}), P_i'(x_{i+1}) = f'(x_{i+1}), 1 \leq i \leq n - 1,$$

$$\begin{aligned} a_i &= f(x_i), \\ b_i &= f'(x_i), \\ c_i &= \frac{3f[x_i, x_{i+1}] - 2f'(x_i) - f'(x_{i+1})}{(x_{i+1} - x_i)} \\ d_i &= \frac{f'(x_i) - 2f[x_i, x_{i+1}] + f'(x_{i+1})}{(x_{i+1} - x_i)^2} \end{aligned}$$

3. Complete the proof of cubic splines outlined in class for all the three possibilities.
4. Write a function file called `wiggle`, with input parameter n, to perform the following tasks.
   (a) Compute n equally spaced points x_k values ($k = 1, \dots, n$) on the interval $-1 \leq x \leq 1$.
   (b) Evaluate $r(x_k)$ where $r : [-1, 1] \to \mathbb{R}$ given by $r(x) = \frac{1}{1+25x^2}$.
   (c) Use the n pairs of $(x_k, r(x_k))$ values to define a $n-1$ degree polynomial interpolant, P_{n-1}.
   (d) Create 100 equally spaced points $\hat{x}_j$ values ($j = 1, \dots, 100$) in the interval $-1 \leq x \leq 1$ and evaluate $P_{n-1}(\hat{x}_k)$.
   (e) Plot $(x_k, r(x_k)), 1 \leq k \leq 10$ with open circles; $(\hat{x}_j, r(\hat{x}_j)), j = 1, \dots, 100$ with solid line; and $(\hat{x}_j, P_{n-1}(\hat{x}_j)), j = 1, \dots, 100$ with dashed line.
   (f) Print the value of $\| r(\hat{x}) - P_{n-1}(\hat{x}) \|_2$.

   Run your `wiggle` function and see if you spot a wiggle effect for $n = 5 : 2 : 15$ and see the behaviour of (f).