1. Manually perform three steps of Euler's method to solve

$$\frac{dy}{dt} = \frac{1}{t+y+1},\ y(0) = 0$$

with $h = 0.2$.

2. Consider

$$\frac{dy}{dt} = t - 2y,\ y(0) = 1.$$

Compute its exact solution. Using `rhs1` and `odeEuler` compute the numerical solution using Euler's method in $[0, 0.6]$ with $h = 0.2$. Use `demoEuler` to compare the numerical solution with the exact solution. Further compute the numerical solution using Euler's method in $[0, 0.6]$ with $h = 0.1$ and $h = 0.05$. Plot all three solutions along with the exact solution.

3. Consider

$$\frac{dy}{dt} = y,\ y(0) = 1\, 0 \le t \le 1$$

Compute its exact solution. Using `rhs2, odeMidpt` compute the numerical solution using the midpoint method in $[0, 1]$ with $h = 0.2$.Use `compEM` to compare this numerical solution with that obtained by Euler's method. For the same accuracy comment on the number of flops required between Euler versus Midpoint method.

4. Consider

$$\frac{dy}{dt} = y,\ y(0) = 1\, 0 \le t \le 1$$

Compute its exact solution. Using `rhs2, odeRK4` compute the numerical solution using the Runge-Kutta method in $[0, 1]$ with $h = 0.2$. Use `compEMRK4` to compare this numerical solutions obtained by Euler, Midpoint and Runge-Kutta (4) method. For the same accuracy comment on the number of flops required between these methods.

5. Use (`odeEuler` or otherwise) Euler's method to with $h = 0.05$ to solve

$$\frac{dy}{dt} = \sqrt{y},\ y(0) = 0,\ 0 \le t \le 2$$

Recompute the solution using `odeMidpt` and `odeRK4`. Plot a comparison of the numerical solution(s) with the exact solution. Does the plot indicate an error in `odeEuler` ?

6. Starting with `odeEuler`, suitably modify it to write an function that will implement Heun's mehthod for an arbitrary first-order ODE. Use your function to solve

$$\frac{dy}{dt} = t - 2y,\ y(0) = 1$$

for $h = 0.2, 0.1, 0.05$ and $h = 0.025$.