Due: Monday, October 29th, 2012
Problems to be turned in: 1,2

1. Suppose A is a subset of $\mathbb{R}$ and c is a limit point of A. Show that there is a sequence $\{x_n\}_{n=1}^{\infty}$ such that $x_n \in A$ and $x_n \to c$.

2. Let S be a subset of $\mathbb{R}$ that contains atleast two points and has the property that: if $x, y \in S$ then $[x, y] \subseteq S$. Show that S is an interval.

3. Let I be an interval. Let $f : I \to \mathbb{R}$ be continuous on I. Let $f(I) = \{f(x) : x \in I\}$. Suppose $k \in \mathbb{R}$ is such that $\inf f(I) \leq k \leq \sup f(I)$. Show that there exists a number $c \in I$ such that $f(c) = k$.

4. For each $x \in \mathbb{R}$, let $g^x : \mathbb{R} \to \mathbb{R}$ be given by $g^x(y) = 1_{(-\infty,x]}(y)$ (that is, $g^x(y) = 1$ if $y \leq x$ and 0 otherwise). Show that g^x is not continuous at x but is continuous otherwise.

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function.
   (a) Suppose $c \in \mathbb{R}$ and $f(c) > 0$. Show that there is a $\delta > 0$ such that $f(x) > 0$ for all $x \in (c - \delta, c + \delta)$
   (b) Consider $Z = \{x \in \mathbb{R} : f(x) = 0\}$. Show that Z contains all its limit points.

6. Find the continuity points of $f : \mathbb{R} \to \mathbb{R}$, when f is given by: (a) $f(x) = \lfloor x \rfloor$ (i.e. greatest integer less than or equal to x), (b) $f(x) = x\lfloor x \rfloor$, and (c) $f(x) = x - \lfloor x \rfloor$. %vfill