Due: Wednesday, October 10th, 2012

Problem to be turned in : 2,4

1. Let $C_1, C_2 \in \mathbb{R} \setminus \{0\}$ and $\{x_n\}_{n=1}^{\infty}$, $\{y_n\}_{n=1}^{\infty}$, $\{z_n\}_{n=1}^{\infty}$be sequences of real numbers. Suppose the series $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ both converge. Show that:

   (a) If $z_n = C_1 x_n + C_2 y_n$ then $\sum_{n=1}^{\infty} z_n$ also converges.

   (b) If $0 \leq z_n \leq C_1 x_n$ then $\sum_{n=1}^{\infty} z_n$ also converges.

2. Suppose $\{x_n\}_{n=1}^{\infty}$, $\{y_n\}_{n=1}^{\infty}$and $\{z_n\}_{n=1}^{\infty}$are sequences of real numbers such that

$$x_n \leq z_n \leq y_n$$

   for all $n \geq 1$, and the series $\sum_{n=1}^{\infty} x_n$, $\sum_{n=1}^{\infty} y_n$ both converge. Does it always imply that $\sum_{n=1}^{\infty} z_n$ converges ?

3. Let $a, b, c \in \mathbb{R}$. Suppose $\{x_n\}_{n=1}^{\infty}$is a sequence of real numbers such that

$$x_n = \frac{a}{n} + \frac{b}{n+1} + \frac{c}{n+2}.$$

   Decide whether the series $\sum_{n=1}^{\infty} x_n$ converges or not.

4. Suppose $\{z_n\}_{n=1}^{\infty}$is a sequence of real numbers. Decide whether the series converges or not in each of the following cases:

$$(i) z_n = \frac{\sqrt{n}}{2n^3 - 1}, (ii) z_n = \left(\frac{n}{2n+1}\right)^n (iii) z_n = \frac{\sqrt{n^2 - n + 1}}{n^3 + 1}$$