Due: Wednesday, August 8, 2012
Problem to be turned in : 5,7.

In this assignment: $\mathbb{N}$ will denote natural numbers, $\mathbb{Q}$ will denote rational numbers, $\mathbb{R}$ will denote real numbers and $\mathbb{C}$ will denote complex numbers.

1. Show that if $0 < a < b$ and $a, b \in \mathbb{R}$ then
$$b^n - a^n < (b-a)nb^{n-1}.$$
2. Let $n \in \mathbb{N}$. Prove that $n^3 + 2n$ is always a multiple of 3.
3. For $x > 0, n \in \mathbb{N}$, let $A = \{t \in \mathbb{R} : t > 0, t^n < x\}$. Let $y = \sup(A)$. Show that $y^n > x$ is not possible.
4. (Rudin: page 21) Let r and x be real numbers. If r is rational $(r = 0)$and x is irrational, prove that $r + x$ and rx are irrational.
5. (Rudin: page 22) Let A be a non-empty set of real numbers which is bounded below. Let
$$-A := \{x \in \mathbb{R} : -x \in A\}.$$
Show that $\inf(A) = -\sup(-A)$.
6. (From Rudin: page 22) Show that the set of all complex numbers $\mathbb{C}$ defined in class is a Field. Decide whether this field can be: (a) ordered set and/or (b) ordered field.
7. (Rudin: page 22) If $z, w, z_i \in \mathbb{C}$ for $i = 1, 2, \ldots, n$ then show that
$$\mid \sum_{k=1}^{n} z_k \mid \leq \sum_{k=1}^{n} \mid z_k \mid .$$
and
$$\mid\mid z \mid - \mid w \mid\mid \leq \mid z - w \mid .$$
8. (From Rudin: page 22) Decide when does the equality hold in Cauchy-Schwarz-Bunyakovski inequality ?